1 **A** Five people can be arranged in a line in $5!$ ways.

2 **C** There are two vowels $\{O, A\}$ and four consonants $\{H, B, R, T\}$. If the arrangement begins with a vowel then there are two choices for the first letter. The remaining five letters can be arranged in $5!$ ways. Using the multiplication principle, there are $2 \times 5! = 240$ arrangements in total.

3 **C** There are five choices for the first digit, four the second, three for the third and two for the fourth. This gives a total of $5 \times 4 \times 3 \times 2$ different numbers.

4 **A** There are six digits in total, of which a group of 3 are alike and a further group of 3 are alike. Therefore, they can be arranged in

$$\frac{6!}{3! \times 3!}$$

different ways.

5 **B** Sam has $2n$ coins in total, of which a group of n are alike and a further group of n are alike. Therefore, they can be arranged in

$$\frac{(2n)!}{n! \times n!} = \frac{(2n)!}{(n!)^2}$$

different ways.

6 **D** Mark is still to select two more flavours out of the nine remaining options. This can be done in ${}^9C_2$ different ways.

7 **D** One must choose two out of four Labour members and two out of five Liberal members. Using the multiplication principle, this can be done in

$${}^4C_2 \times^5 C_2$$

different ways

8 **D** A set with ten elements (friends!) has 2^10 subsets (of friends). This includes the empty set. However, because we are inviting at least one friend, the empty set must be excluded. This leaves $2^10 - 1$ subsets.

9 **A** Create 3 holes for each of the different utensils, (K,F,S). Clearly selecting 9 items and placing them in their corresponding hole may not be sufficient, as you could get 3 of each type. However, if 10 are selected then, since $10 = 3 \times 3 + 1$, by the generalised pigeonhole principle there must be some hole with at least 4 utensils. Therefore the smallest number of items is 10.

10 **E** There are three possible remainders when a number is divided by 3. Label three holes with each of these remainders:

$$\boxed{0}\ \boxed{1}\ \boxed{2}$$

If 15 integers are written on the board, then placed in their corresponding box, then this may not be sufficient - you could get 5 of each remainder. However, if 16 integers are written on the board then, since $16 = 5 \times 3 + 1$, by the generalised pigeonhole principle there must be some hole with at least 6 integers. Therefore the smallest number of integers is 16.

11 **B** Let A and B be the sets comprising of multiples of 2 and 5 respectively. Clearly $A \cap B$ consists of the multiples of 2 and 5, that is, multiples of 10. Therefore, $|A| = 30, |B| = 12$ and $|A \cap B| = 6$. We then use the Inclusion Exclusion Principle to find that,

$$\begin{aligned} |A \cup B| &= |A| + |B| - |A \cap B| \\ &= 30 + 12 - 6 \end{aligned}$$

$= 36.$

12 **E** All are true except the last option. For if m and n are odd then so is mn. Therefore $mn + 1$ will be even.

13 **E**

- Item A is true. Since n is divisible by 12, it will be divisible by 3. Therefore, $m \times n$ will be divisible by 3.
- Item B is true. Since $m = 4j$ and $n = 12k$ we know that $m \times n = 48jk$, which is divisible by 48.
- Item C is true. Since $m = 4j$ and $n = 12k$ we know that $m + n = 4j + 12k = 4(j + 3k)$, which is divisible by 4.
- Item D is true. Since $m \times n$ is divisible by 48, it follows that $m^2 n$ will also be divisible by 48.
- Item E may be false. For example, $n = 12$ is divisible by 12 and $m = 16$ is divisible by 4 and yet n is not divisibly by m.

14 **D**

- Item A is false. If $m = 3$ and $n = 2$ then $mn = 6$ is even, though m is not even.
- Item B is false. If $m = 1$ and $n = 3$ then $m + n = 4$ is even, even though neither m nor n is even.
- Item C is false. If $m = 1$ and $n = 2$ then $m + n = 3$ is odd, while $mn = 2$ is even.
- Item D is true. If mn is odd then both m and n are odd. Therefore $m + n$ is even.
- Item E is false. Note that $m + n$ and $m - n$ will both be odd, or both be even.

15 **C** To form the converse, we switch the hypothesis (If n is even) and the conclusion (then $n + 3$ is odd). This gives "If $n + 3$ is odd, then n is even".

16 **E**

- Item A is true. If $a > b$, then $a - b > 0$. Therefore $\dfrac{1}{a-b} > 0$.
- Item B is true. If $a > b$ then $\dfrac{a}{b} - \dfrac{b}{a} = \dfrac{a^2 - b^2}{ab} > 0$.
- Item C is true. If $a > b$ then $a + b > b + b = 2b$.
- Item D is true. If $a > b$ then $a + 3 > b + 2$.
- Item E may be false. For example, if $a = 3$ and $b = 2$ then $a > b$ while $2a = 6 = 2b$.

17 **E** Since,

$$\begin{aligned} mn - n &= 12 \\ n(m-1) &= 12 \end{aligned}$$

Clearly $n = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$. And for each of these twelve values of n we can easily find a corresponding value of m.

18 **B** Since $9n^2 - 4 = (3n - 2)(3n + 2)$, the number will always be composite, unless $3n - 2 = 1$. This implies that $n = 1$, in which case $9n^2 - 4 = 5$. So there is only one such value of n.

19 **B**

- Item A is true. For any four consecutive numbers, two will be even, two will be odd. So the sum will be even.
- Item B may be false. For example, $1 + 2 + 3 + 4 = 10$ is not divisible by 4.
- Item C is true. For any four consecutive numbers, one will be divisible by 3.

- Item D is true. For any four consecutive numbers, one will be divisible by 4, and another will be divisible by 2. Therefore the product will be divisible by 8.
- Item E is true. For any four consecutive numbers, one will be divisible by 4, and another will be divisible by 2. Furthermore, one number will be divisible by 3. Therefore the product will be divisible by 24.

20 D

From $\triangle ADB$
$x^2 + 36 = AC^2$
From $\triangle BCA$
$x^2 + 36 + x^2 = 100$
$2x^2 = 64$
$x = 4\sqrt{2}$

21 D From $\triangle ABC$
$9 + 36 = BC^2$
From $\triangle DBC$
$DB^2 = 9 + 45$
$DB = 3\sqrt{6}$

22 A $\triangle ADE \sim \triangle ACB$

$$\therefore \frac{DE}{12} = \frac{AE}{AB}$$

$DE = 6$

23 E $\angle SRQ = x°$ (isosceles triangle)
$\angle RSQ = (138 - y)°$ (co-interior angle)
$2x + 138 - y = 180$ (angle sum of triangle)
$\therefore 2x - y = 42$

24 C Convert both to mm.s

The scale is $45 : 17\,100$

Divide both parts of the ratio by 45.

$\text{Scale} = 1 : 380$

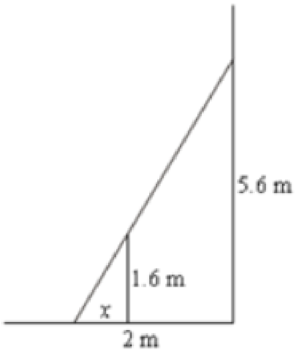
25 B

$$\begin{aligned}
\frac{AG}{DG} &= \frac{15}{5} = 3 \\
AG &= 3DG = 18 \\
\therefore \quad AE &= 18 - 9 = 9 \\
\frac{BG}{FG} &= \frac{AG}{EG} \\
\frac{FG+6}{FG} &= \frac{18}{9} = 2 \\
FG + 6 &= 2FG \\
\therefore \quad FG &= 6 \\
\frac{x}{FG} &= \frac{6}{9} = \frac{2}{3} \\
x &= \frac{2}{3}FG \\
&= \frac{2}{3} \times 6 = 4
\end{aligned}$$

26 E Write both in cm.

The ratio is $8 : 320 = 1 : 40$

27 A



Use similar triangles.

$$\frac{x}{2} = \frac{1.6}{5.6}$$
$$= \frac{16}{56} = \frac{2}{7}$$
$$x = \frac{4}{7}$$
$$\approx 0.57 \text{ m}$$

28 C

$$\frac{DE}{AB} = \frac{10}{4} = 2.5$$
$$DE = 2.5 \times AB$$
$$\text{Area} = 2.5^2 \times 24$$
$$= 150 \text{ cm}^2$$

29 C

$$\frac{A+B}{A} = \frac{5^3}{3^3} = \frac{125}{27}$$
$$\frac{A+49}{A} = \frac{125}{27}$$
$$27A + 49 \times 27 = 125A$$
$$98\ A = 49 \times 27$$
$$A = \frac{49 \times 27}{98}$$
$$= \frac{27}{2} = 13.5 \text{ cm}^3$$

30 E

$$\frac{2}{3}\pi R^3 : \frac{1}{3}\pi^2 r^2 = 27 : 4$$
$$2R^3 : r^3 = 27 : 4$$
$$R^3 : r^3 = \frac{27}{2} : 4$$
$$= 27 : 8$$
$$R : r = \sqrt[3]{27} : \sqrt[3]{8}$$
$$R : r = 3 : 2$$

31 D

$$KO : KN = 1 : 3$$
$$\frac{\text{area } KOP}{\text{area } MLK} = \frac{1}{9}$$
$$\frac{\text{area } KOP}{\text{area } (MLK + MNK)} = \frac{1}{9} \times \frac{1}{2}$$
$$= \frac{1}{18}$$
$$\frac{\text{area } KOP}{\text{area } KLMN} = \frac{1}{18}$$

32 **B** The angle subtended at the top of the circle by $QT = \frac{150}{2} = 75^\circ$.

By the alternate segment theorem, $\angle QTS = 75^\circ$.

33 **C** Join LN.

Using the alternate segment theorem, $\angle MLN = 40^\circ$.

In triangle LMN, $ML = MN$

$$\begin{aligned} \therefore\ \angle MNL &= \angle MLN = 40^\circ \\ \angle LMN &= 180 - 40 - 40 \\ &= 100^\circ \end{aligned}$$

34 **B** Using the alternate segment theorem,

$$\begin{aligned} \angle ZYX &= \angle ZXT \\ \angle ZXT &= \angle ZXY \\ \therefore\ \angle ZYX &= \angle ZXY \end{aligned}$$

Triangle ZXY is isosceles, with $YZ = XZ$.

35 **D**

$$\begin{aligned} \angle QOS &= 180 - 70 \\ &= 110^\circ \\ \text{Reflex } \angle QOS &= 360 - 110 \\ &= 250^\circ \\ \angle QRS &= \frac{250}{2} \\ &= 125^\circ \end{aligned}$$

36 **C** Let the diameter be CE.

If $AD = DB = 4$ cm, then

$$\begin{aligned} AD \cdot DB &= CD \cdot DE \\ 4 \times 4 &= 2(2r - 2) \\ 2r - 2 &= 8 \\ 2r &= 10 \\ r &= 5 \text{ cm} \end{aligned}$$

37 **C** Join AB.

Since $TA = TB$, $\angle TBA = 45^\circ$.

Since AC is perpendicular to tangent TA, it must be a diameter.

$$\begin{aligned} \therefore\ \angle CBA &= 90^\circ \\ \angle TBC &= 90 + 45 \\ &= 135^\circ \end{aligned}$$

TB is parallel to AC, since co-interior angles BTA and CAT are supplementary.

$$\begin{aligned} \therefore\ \angle BCA &= 180 - 135 \\ &= 45^\circ \end{aligned}$$

38 **B**

$$\begin{aligned} \angle RTP &= 30^\circ \text{(alternate segments)} \\ \angle TRS &= 40 + 30 \\ &= 70^\circ \text{(exterior angle of } RTP\text{)} \\ \angle RTS &= 180 - 70 - 30 \\ &= 80^\circ \end{aligned}$$

39 **D**

$$
\begin{aligned}
AB &= AC \\
\therefore \angle ADC &= 60^\circ \\
\angle ACD &= 180 - 60 - 50 \\
&= 70^\circ (\text{angle sum of } RTS) \\
\angle ABD &= 180 - 70 \\
&= 110^\circ
\end{aligned}
$$